## HYPOTHESIS TESTING WITH Z TESTS

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## Review: Standardization

Allows us to easily see how one score (or sample) compares with all other scores (or a population).


## CDC Example: Jessica

Jessica is 15 years old and 66.41 in . tall For 15 year old girls, $\mu=63.8, \sigma=2.66$

$$
z=\frac{(X-\mu)}{\sigma}=\frac{(66.41-63.8)}{2.66}=0.98
$$

| $z$ | \% BETWEEN MEAN AND $z$ |
| :---: | :---: |
| . | . |
| $\cdot$ | $\cdot$ |
| 0.97 | 33.40 |
| 0.98 | 33.65 |
| 0.99 | 33.89 |
| 1.00 | 34.13 |
| 1.01 | 34.38 |
| 1.02 | 34.61 |

## CDC Example: Jessica

$\square$ 1. Percentile: How many 15 year old girls are shorter than Jessica?
$\square 50 \%+33.65 \%=83.65 \%$


## CDC Example: Jessica

$\square$ 2. What percentage of 15 year old girls are taller than Jessica?
$\square 50 \%-33.65 \%$ OR $100 \%-83.65 \%=16.35 \%$


## CDC Example: Jessica

$\square$ 3. What percentage of 15 year old girls are as far from the mean as Jessica (tall or short)?
口 $16.35 \%+16.35 \%=32.7 \%$


## CDC Example: Manuel

$\square$ Manuel is 15 years old and 61.2 in. tall
$\square$ For 15 year old boys, $\mu=67, \sigma=3.19$

$$
z=\frac{(X-\mu)}{\sigma}=\frac{(61.2-67)}{3.19}=-1.82
$$

$\square$ Consult z table for $1.82 \rightarrow 46.56 \%$

## CDC Example: Manuel

$\square$ 1. Percentile
$\square$ Negative z, below mean: 50\% - 46.56\% = 3.44\%


## CDC Example: Manuel

$\square$ 2. Percent Above Manuel

- 100\%-3.44\% = 96.56 \%



## CDC Example: Manuel

$\square$ 3. Percent as extreme as Manuel
$\square 3.44 \%+3.44 \%=6.88 \%$


## Percentages to z Scores

SAT Example: $\mu=500, \sigma=100$
$\square$ You find out you are at $63^{\text {rd }}$ percentile
$\square$ Consult $z$ table for $13 \% \rightarrow z=.33$

$$
x=.33(100)+500=533
$$



## z Table and Distribution of Means

$\square$ Remember that if we use distribution of means, we are using a sample and need to use standard error.
$\square$ How do UMD students measure up on the GRE?
$\square \mu=554, \sigma=99$

$$
M=568, N=90
$$

$$
\square \mu_{M}=\mu=554 \quad \sigma_{M}=\frac{\sigma}{\sqrt{N}}=\frac{99}{\sqrt{90}}=10.436
$$

## UMD \& GRE Example

$$
z=\frac{\left(M-\mu_{M}\right)}{\sigma_{M}}=\frac{(568-554)}{10.436}=1.34
$$

$\square$ Consult $z$ table for $z=1.34 \rightarrow 40.99 \%$
$\square 50 \%+40.99 \%=90.99 \%$


## Assumptions of Hypothesis Testing

1. The DV is measured on an interval scale
2. Participants are randomly selected
3. The distribution of the population is approximately normal

Robust: These hyp. tests are those that produce fairly accurate results even when the data suggest that the population might not meet some of the assumptions.

- Parametric Tests
- Nonparametric Tests


## Assumptions of Hypothesis Testing

## TABLE 8-2. THE THREE ASSUMPTIONS FOR HYPOTHESIS TESTING

We must be aware of the assumptions for the hypothesis test that we choose, and we must be cautious in choosing to proceed with a hypothesis test even though our data may not meet all of the assumptions. Note that in addition to these three assumptions, for many hypothesis tests, including the $z$ test, the independent variable must be nominal.

THE THREE ASSUMPTIONS

1. Dependent variable is measured on an interval scale.
2. Participants are randomly selected.
3. Population distribution is approximately normal.

BREAKING THE ASSUMPTIONS
Usually OK if the data are not clearly nominal or ordinal.
OK if we are cautious about generalizing.
OK if the sample includes at least 30 scores.

## Testing Hypotheses (6 Steps)

1. Identify the population, comparison distribution, inferential test, and assumptions
2. State the null and research hypotheses
3. Determine characteristics of the comparison distribution

- Whether this is the whole population or a control group, we need to find the mean and some measure of spread (variability).


## Testing Hypotheses (6 Steps)

4. Determine critical values or cutoffs

- How extreme must our data be to reject the null?
- Critical Values: Test statistic values beyond which we will reject the null hypothesis (cutoffs)
- p levels (a): Probabilities used to determine the critical value

5. Calculate test statistic (e.g., z statistic)
6. Make a decision

- Statistically Significant: Instructs us to reject the null hypothesis because the pattern in the data differs from what we would expect by chance alone.


## The z Test: An Example

$$
\mu=156.5, \sigma=14.6, M=156.11, N=97
$$

. Populations, distributions, and assumptions

- Populations:

1. All students at UMD who have taken the test (not just our sample)
2. All students nationwide who have taken the test

- Distribution: Sample $\rightarrow$ distribution of means
- Test \& Assumptions: $\boldsymbol{z}$ test

1. Data are interval
2. We hope random selection (otherwise, less generalizable)
3. Sample size $>30$, therefore distribution is normal

## The z Test: An Example

2. State the null and research hypotheses

$$
\begin{aligned}
& \mathrm{H}_{0}: \mu_{1} \leq \mu_{2} \\
& \mathrm{H}_{1}: \mu_{1}>\mu_{2}
\end{aligned}
$$

OR

$$
\begin{aligned}
& \mathrm{H}_{0}: \mu_{1}=\mu_{2} \\
& \mathrm{H}_{1}: \mu_{1} \neq \mu_{2}
\end{aligned}
$$

## The z Test: An Example

3. Determine characteristics of comparison distribution.

- Population: $\mu=156.5, \sigma=14.6$
- Sample: $M=156.11, N=97$

$$
\sigma_{M}=\frac{\sigma}{\sqrt{N}}=\frac{14.6}{\sqrt{97}}=1.482
$$

## The z Test: An Example

4. Determine critical value (cutoffs)

- In Behavioral Sciences, we use p $=.05$
- $p=.05=5 \% \rightarrow 2.5 \%$ in each tail
- $50 \%-2.5 \%=47.5 \%$
- Consult $z$ table for $47.5 \% \rightarrow z=1.96$



## The z Test: An Example

5. Calculate test statistic

$$
z=\frac{\left(M-\mu_{M}\right)}{\sigma_{M}}=\frac{(156.11-156.5)}{1.482}=-0.26
$$

6. Make a Decision


## Increasing Sample Size

$\square$ By increasing sample size, one can increase the value of the test statistic, thus increasing probability of finding a significant effect

## Increasing Sample Size

$\square$ Example: Psychology GRE scores
Population: $\mu=554, \sigma=99$
Sample: $M=568, \underline{N=90}$

$$
\begin{gathered}
\sigma_{M}=\frac{\sigma}{\sqrt{N}}=\frac{99}{\sqrt{90}}=10.436 \\
Z=\frac{\left(M-\mu_{M}\right)}{\sigma_{M}}=\frac{(568-554)}{10.436}=1.34
\end{gathered}
$$

## Increasing Sample Size

$\square$ Example: Psychology GRE scores
Population: $\mu=554, \sigma=99$
Sample: $M=568, \underline{N=200}$

$$
\begin{gathered}
\sigma_{M}=\frac{\sigma}{\sqrt{N}}=\frac{99}{\sqrt{200}}=7.00 \\
Z=\frac{\left(M-\mu_{M}\right)}{\sigma_{M}}=\frac{(568-554)}{7.00}=2.00
\end{gathered}
$$

